

Algebra 2

Week # 2

3-1 Additional Practice

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Graphing Polynomial Functions

Write each polynomial in standard form. Then classify it by degree and by number of terms.

1. 4x + x + 2 **2.** $1 - 2s + 5s^4$

Use the leading coefficient and degree of the polynomial function to determine the end behavior of the graph.

3. $f(x) = -2x^4 - x^3 + 5x^2 - 2x + 3$ **4.** $f(x) = 4x^2 + 4x - 6$

Sketch the graph using the clues listed. Identify the turning points and x-intercepts.

5. f(x) is negative on the intervals $(-\infty, -5)$ and (-1, 3)

- f(x) is positive on the intervals (-5, -1) and $(3, \infty)$
- f(x) is increasing on the interval $(-\infty, -3.5)$ and $(1.25, \infty)$
- f(x) is decreasing on the interval (-3.5, 1.25)
- 6. Keegan is printing and selling his original design on *t*-shirts. He has concluded that for *x* shirts, in thousands sold his total profits will be p(x) = dollars, in thousands will be earned. How many *t*-shirts (rounded to the nearest whole number) should he print in order to make maximum profits? What will his profits rounded to the nearest whole dollar be if he prints that number of shirts?
- **7.** The table at the right shows data representing a polynomial function.
 - a. What is the degree of the polynomial function?
 - **b.** What are the second differences of the *y*-values?

c. What are the differences when they are constant?

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| | , |
|----|------|
| -3 | -999 |
| -2 | -140 |
| -1 | -7 |
| 0 | 0 |
| 1 | 1 |
| 2 | 116 |
| 3 | 945 |



enVision Algebra 2

3-2 Additional Practice

Adding, Subtracting and Multiplying Polynomials

Add or subtract the polynomials.

- 1. $(4x^3 + 2x + 2x^2 8) + (2x^3 + x^2 + 9)$
- **2.** $(y^3 + 6x^2y^2 4xy 8) (2y^3 7x^2y^2 2xy y + 8)$
- **3.** $(9a^{3}b + 6ab 4) (10a^{3}b 6a^{2}b^{2} 6)$

Multiply the polynomials.

4. $-2cd(5c^2 - 5cd - d^2)$ **5.** $(-2b + 4)(5b^2 - 4b + 2)$

Are the following polynomial sets open or closed?

6. $(x^2 + x - 4) - (x^2 + x + 8)$ **7.** (2 - x)(1 + 3x) **8.** (5b - 3c)(7b - 3c)

Write a Polynomial Function.

- **9.** Write and simplify a polynomial expression to find the area of 4 circles. Each circle has a radius of (4a 6).
- **10.** If the length of a rectangle in terms of x centimeters is $5x^2 + 4x 4$ and its width is $3x^2 + 2x + 6$ centimeters, what is the perimeter of the rectangle? Simplify.

Compare the maximum values and the end behavior of the functions of *f* and *g*.





3-3 Additional Practice

Polynomial Identities

Prove the polynomial identity.

1.
$$x^2 - y^2 = (x - y)(x + y)$$
 2. $(x^4 - y^4) = (x^2 + y^2)(x + y)(x - y)$

Use polynomial identities to multiply the polynomial.

3. (3x + 9) (3x - 9) **4.** $(-6x^2 + 7y^3)^2$ **5.** $(8x^4 + 5y^3)^2$

Use polynomial identities to factor the polynomial.

- **6.** $n^6 25m^4$ **7.** $16x^{12} 64y^4$ **8.** $b^2 36c^4$
- **9.** $25x^6 100y^4$ **10.** $225x^6 y^{10}$

Expand the equations using Pascal's Triangle and the Binomial Theorem.

11.
$$(x + 0.5)^3$$
 12. $(s + 4t)^6$

Use Pascal's Triangle to expand the equations below.

13.
$$(3a - 3b)^4$$
 14. $(3m - 2n)^5$ **15.** $(a - 4)^5$

16. A rectangular lawn has an area of $a^3 - 125$. Use the difference of cubes to find out the dimensions of the rectangle.

3-4 Additional Practice

Dividing Polynomials

Divide using long division.

1. $(x^2 - 13x - 48)$ $\div (x + 3)$ **2.** $(x^3 + 5x^2 - 3x - 1)$ $\div (x - 1)$ **3.** $(3x^3 - x^2 - 7x + 6)$ $\div (x + 2)$

Divide using synthetic division.

- **4.** $(x^3 8x^2 + 17x 10)$ $\div (x - 5)$ **5.** $(x^3 + 5x^2 - x - 9)$ $\div (x + 2)$ **6.** $(2x^4 + 7x^3 - 11x^2 + 21x + 5) \div (x + 5)$
- 7. Verify the Remainder Theorem if $P(x) = x^3 5x^2 7x + 25$ is divided by (x 5). Explain.

Determine whether each binomial is a factor of $x^3 + 3x^2 - 10x - 24$.

- **8.** *x* + 4 **9.** *x* 3 **10.** *x* + 6
- 11. The volume, in cubic inches, of a rectangular box can be expressed as the product of its three dimensions: $V(x) = x^3 16x^2 + 79x 120$. The length is x 8. Find linear expressions with integer coefficients for the width and height. The width is greater than the height.
- 12. What does it mean if P(-4) for the polynomial function $P(x) = x^3 + 11x^2 + 34x + 24$ equals zero?

3-5 Additional Practice

Zeros of Polynomial Functions

Sketch the graph of the function by finding the zeros. List the zeros.

Find the zeros of each function and describe the behavior of the graph of the function at each zero.

3. $x^3 - 8x^2 + 18x$ **4.** $x^3 + x^2 - 3x + 1$

Determine all the real and complex zeros of each polynomial function.

- **5.** $f(x) = x^3 7x^2 + 4x 28$ **6.** $f(x) = x^3 - x^2 - 2x + 8$
- 7. A company that sells toys models their profit with the function $P(x) = -4x^3 + 32x^2 64$. Their profit *P*, in thousands of dollars, is a function of the number of toys sold *x* measured in hundreds. What do the key features of the graph reveal about the profits? What is the maximum profit the company can make?

Solve each inequality.

- **8.** $x^3 27x < 0$ **9.** $x^3 + 9x^2 10x > 0$
- **10.** Use your graphing calculator to determine if f(x) = (x 1)(x 6)(x + 3) is the correct factorization of $f(x) = x^3 + 7x^2 + 4x 12$. Explain.

3-6 Additional Practice

Theorems About Roots of Polynomial Equations

List all the possible rational solutions for each equation.

1. $2x^2 + 5x + 3 = 0$ **2.** $2x^4 - 18x^2 + 5 = 0$ **3.** $4x^3 - 12x + 9 = 0$

List all the real and complex roots of each of the following functions.

- **4.** $x^3 + x^2 x + 2 = 0$ **5.** $x^3 - 2x^2 + 4x - 8 = 0$ **6.** $x^5 - 3x^4 - 8x^3 - 8x^2 - 9x - 5 = 0$
- **7.** What is the equation of a quadratic function *P* with rational coefficients that has a zero of 3 + 7i?
- **8.** What is the equation of a polynomial function, *R*, with rational coefficients that have a zero of $4 + \sqrt{5}$ and 3i?
- **9.** A section of roller coaster can be modeled by the function: $f(x) = x^5 - 5x^4 - 31x^3 + 113x^2 + 282x - 360$. A walkway bridge will be placed at one of the zeros. What are the possible locations for the walkway bridge?
- 10. A shed in the shape of a rectangular prism measures x feet high, x + 6.5 feet wide, and is x 4 feet deep. The volume of the shed is given by the function $v(x) = x^2 + 2.5x 26$. What is the height, width, and depth of the shed, in feet, if the volume is 990 ft³?
- **11.** Suppose a cubic polynomial, f, has two rational roots c and d and one irrational root which is a conjugate pair $a + \sqrt{b}$, where a and b are rational numbers. Does f have rational coefficients? Explain.

4-1 Additional Practice

Inverse Variation and the Reciprocal Function

2.

Do the tables below represent a direct variation or an inverse variation? Explain.

| 1. | x | у |
|----|----|----|
| | 2 | 10 |
| | 4 | 5 |
| | 5 | 4 |
| | 20 | 1 |

| x | у |
|---|----|
| 1 | 6 |
| 2 | 12 |
| 5 | 30 |
| 7 | 42 |

| 3. | x | y |
|----|-----|------|
| | 0.2 | 25 |
| | 0.5 | 62.5 |
| | 2 | 250 |
| | 3 | 375 |

Suppose x and y vary inversely. Write an equation that models each inverse variation. Find y when x = 10.

4. x = 7 when y = 2 **5.** x = 4 when y = 0.2 **6.** x = 2 when y = 5

Graph each function. Identify the asymptotes of each graph and state the domain and the range of each function.

7.
$$f(x) = \frac{12}{x}$$

Asymptotes: Domain:

Domain:

Range:

Range:

Domain:

- **9.** The length of a pipe in a panpipe ℓ , in ft, is inversely proportional to its pitch p, in hertz. The inverse variation is modeled by the equation $p = \frac{497}{\ell}$. Find the length of pipe required to produce a pitch of 220 Hz.
- **10.** From the table of values, how can you determine that the data do not represent an inverse variation?

| x | -4 | -2 | 2 | 4 | 6 | 8 |
|---|-----|-----|-----|----|----|----|
| y | 100 | 100 | 100 | 50 | 25 | 20 |



4-2 Additional Practice



Graphing Rational Functions

Use long division to rewrite each rational function. Sketch the graph and identify the asymptotes.

1.
$$f(x) = \frac{2x}{x+1}$$





Identify the vertical and horizontal asymptotes of each rational function.

3.
$$f(x) = \frac{2x^2}{4x^2 - 1}$$

 $f(x) = \frac{2x^2 + 10x + 12}{x^2 - 9}$

Graph each function. Label all the horizontal and vertical asymptotes.



- 7. You start a business typing papers for students and spend \$3,500 on a computer and office furniture. You estimate additional costs at \$0.02 per page. Write a rational function to model the total average cost per page for the first year.
- 8. The graph of a rational function has vertical asymptotes at x = -3 and x = 3 and a horizontal asymptote at y = 1. Write a function that has those attributes. Then graph the function to verify that it is correct.

4-3 Additional Practice

Multiplying and Dividing Rational Expressions

Write an equivalent expression. Specify the domain.

1. $\frac{4x+6}{2x+3}$ **2.** $\frac{3x^2-12}{x^2-x-6}$ **3.** $\frac{x^2+13x+40}{x^2-2x-35}$

What is the simplified form of each rational expression? Specify the domain.

| 4. $\frac{2x^2 + 11x + 5}{x^2 + 11x + 5}$ | 5. $\frac{6x^2 + 5xy - 6y^2}{x^2 - 5xy - 6y^2}$ | 6. $\frac{x^2 + 3x - 18}{2}$ |
|---|---|------------------------------|
| $3x^2 + 17x + 10$ | $3x^2 - 5xy + 2y^2$ | <i>x</i> ² – 36 |

Find the product and the domain.

7. $\frac{5a}{5a+5} \cdot (10a+10)$ 8. $\frac{x^2-5x}{x^2-3x} \cdot \frac{x+3}{x-5}$ 9. $\frac{5y-20}{3y+15} \cdot \frac{7y+35}{10y+40}$

Find the quotient and the domain.

- **10.** $\frac{7x^4}{24y^5} \div \frac{21x}{12y^4}$ **11.** $\frac{y^2 49}{(y 7)^2} \div \frac{5y + 35}{y^2 7y}$ **12.** $\frac{y^2 5y + 4}{y^2 1} \div \frac{y^2 9}{y^2 + 5y + 4}$
- **13.** A farmer must decide whether to build a cylindrical grain silo with radius *r*, or a rectangular grain silo with width *r* and length 2*r*. Both silos have the same height *h*. Which has the greater ratio of volume to surface area? Explain.
- 14. How do you know what values to exclude from the domain?

4-4 Additional Practice

Adding and Subtracting Rational Expressions

Find the LCM for each group of expressions.

1. $2x^2 - 8x + 8$ and
 $3x^2 + 27x - 30$ 2. $4x^2 + 12x + 9$ and
 $4x^2 - 9$ 3. $2x^2 - 18$ and
 $5x^3 + 30x^2 + 45x$

Find the sum.

4.
$$\frac{6y-4}{y^2-5} + \frac{3y+1}{y^2-5}$$

5. $\frac{x+2}{x^2+4x+4} + \frac{2}{x+2}$
6. $\frac{4}{x^2-25} + \frac{6}{x^2+6x+5}$

Find the difference.

7.
$$-\frac{2}{n+4} - \frac{n^2}{n^2 - 16}$$
 8. $\frac{3}{8x^3y^3} - \frac{1}{4xy}$ 9. $\frac{y}{4y+8} - \frac{1}{y^2 + 2y}$

Simplify.

- **10.** $\frac{\frac{2}{x}+6}{\frac{1}{y}}$ **11.** $\frac{\frac{x+3}{x-3}}{\frac{x^2-9}{3x-9}}$ **12.** $\frac{\frac{5}{x+3}}{2+\frac{1}{x+3}}$
- **13.** At an amusement park, guests have to take either a train or a boat 4 miles from the parking lot to the front entrance and then back when they leave the park. The train goes 10 mph faster than the boat. Abdul takes the train into the park and the boat on his way back. The boat goes an average speed of 20 mph. How long did the round trip take?
- **14.** What is the disadvantage of adding two rational expressions using a common denominator that is not the least common denominator?